圆切割四分之一

小圆滚滚

1 正方形一刀切割四分之一

设正方形边长为1，取刀切入点的横坐标为a，则边长1=a+(1-a)，被切掉的三角形面积为$S=\frac{1}{2}a^2$，若s为面积1的四分之一，则$\frac{1}{2}a^2=\frac{1}{4}$.

$\therefore a=\frac{1}{\sqrt{2}}=\frac{1}{2^{\frac{1}{2}}}=2^{-\frac{1}{2}}=\frac{2^{\frac{1}{2}}}{2^1}=\frac{\sqrt{2}}{2}=\frac{1.4142135623730951}{2}=0.7071067811865476$

即蓝色直角三角形的边长是正方形边长的十分之七。

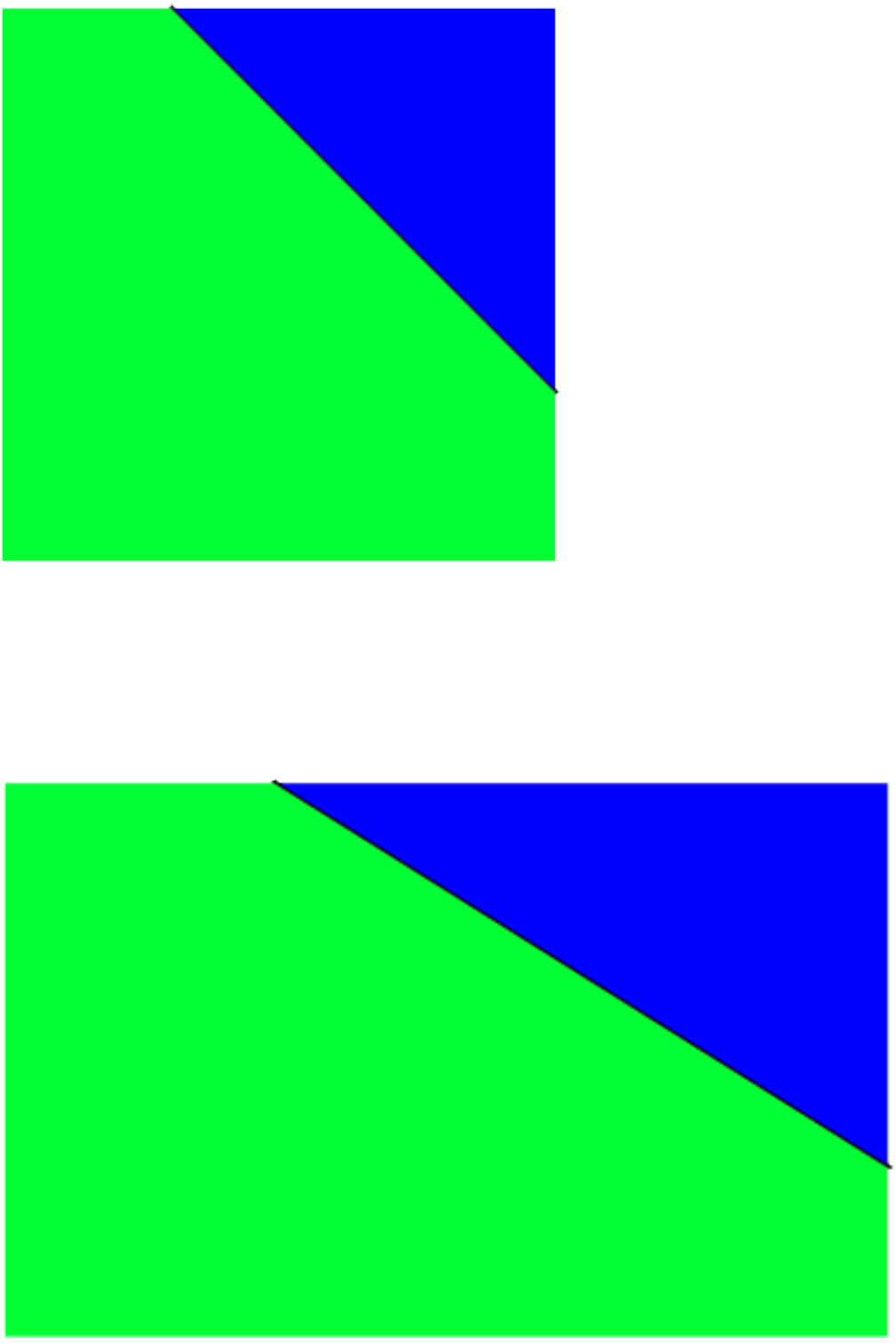

图 1: 正方形一刀切掉四分之一

长方形可以拉伸正方形，比例不变。

2 圆形一刀切四分之一

已知公式：

$360° = 2\pi$弧度

$sin\theta = 2sin\frac{\theta}{2}cos\frac{\theta}{2}$

红色面积定义为：S_1，扇形面积定义为：S_2，扇形面积中包含的蓝色三角形面积为S_3，圆面积定义为S，则：

扇形面积与圆整体的面积的比等于弧长与圆周长的比，等于夹角与2π的比：

$\frac{S_2}{\pi r^2} = \frac{x}{2\pi r} = \frac{\theta}{2\pi}$

$\Rightarrow S_2 = \frac{\theta r^2}{2}$

$S_1 = S_2 - S_3 = \frac{\theta r^2}{2} - \frac{1}{2} \times r \cdot sin\frac{\theta}{2} \times r \cdot cos\frac{\theta}{2} \times 2 = \frac{r^2(\theta - 2sin\frac{\theta}{2}cos\frac{\theta}{2})}{2} = \frac{r^2(\theta - sin\theta)}{2}$

若使：

$\frac{S_1}{S} = \frac{1}{4}$，其中S面积等于单位圆1

则：

$\frac{r^2(\theta - sin\theta)}{2} = \frac{1}{4}$

又$\because \pi r^2 = 1$

$\Rightarrow r = \sqrt{\frac{1}{\pi}}$，代入得：

$\theta - sin\theta = \frac{\pi}{2}$

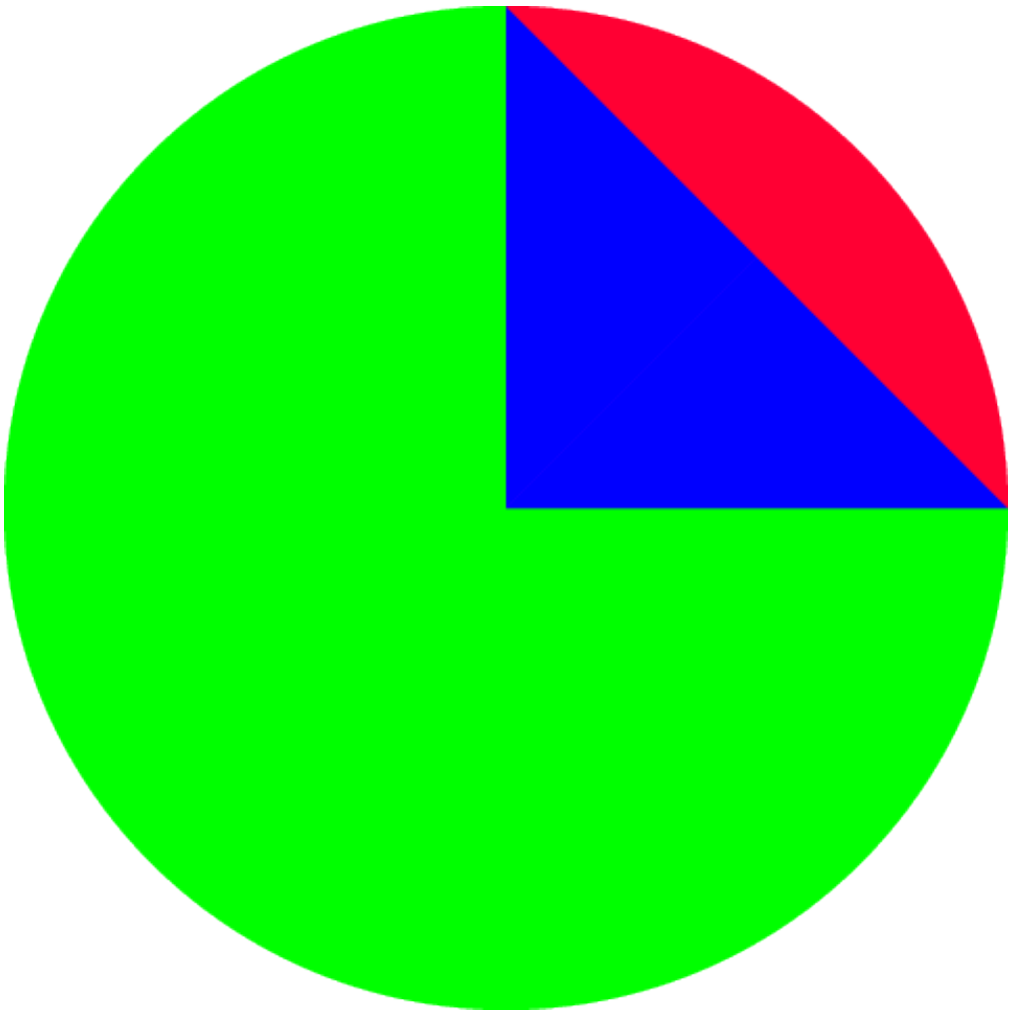

图 2: 圆一刀切掉四分之一示意

3 函数几何含义

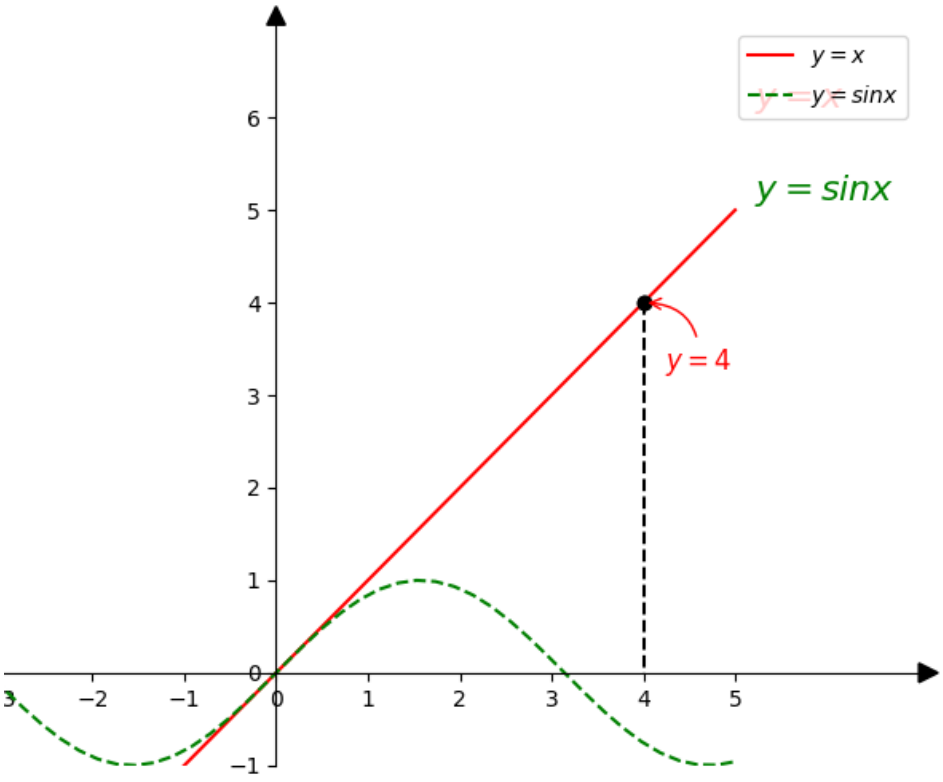

图 3: 两函数相减差值等于$\frac{\pi}{2}$

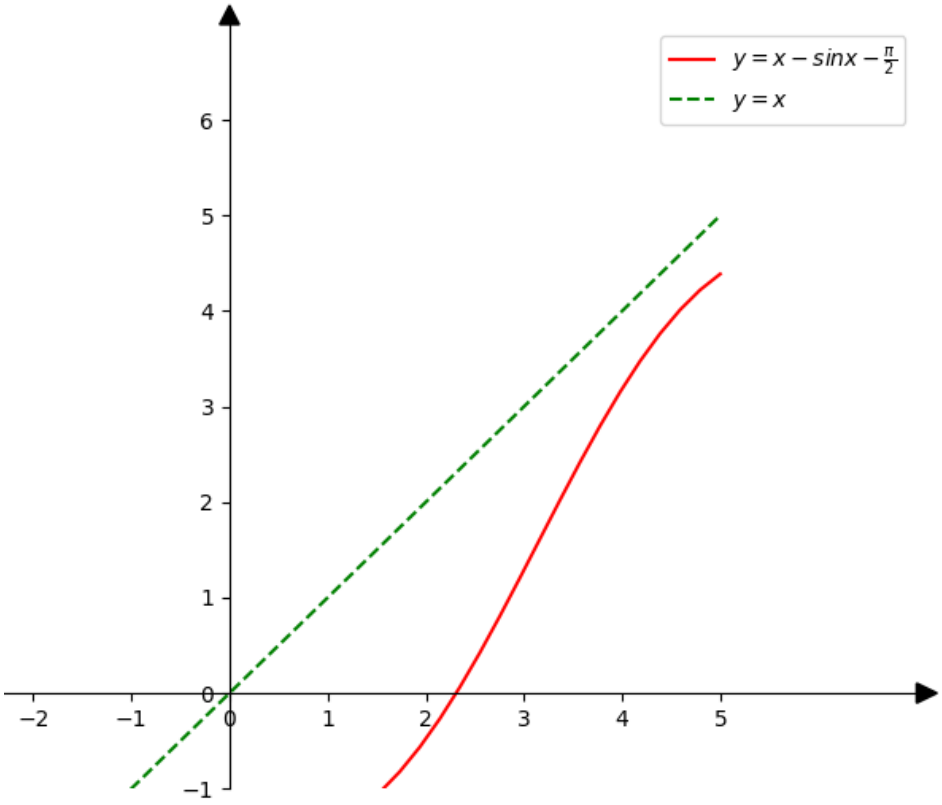

图 4: 函数$y = x - \frac{sinx}{2} - \frac{\pi}{2}$

4 利用牛顿迭代法计算函数在x轴上的根

```
import math

"""定义
 f(x)
"""


def f(x):
    return x - math.sin(x) - math.pi / 2


"""定义
 f'(x)
"""

```

```python
def fd(x):
    return 1 - math.cos(x)


def newtonMethod(n, assum):
    time = n
    x = assum
    Next = 0
    A = f(x)
    B = fd(x)
    print('A = ' + str(A) + ',B = ' + str(B) + ',time = ' + str(time))
    if f(x) == 0.0:
        return time, x
    else:
        Next = x - A / B
        print('Next x = ' + str(Next))
    if abs(A - f(Next)) < 1e-6:  # 设置迭代跳出条件，同时输出满足f(x) = 的0值x
        print('Meet f(x) = 0,x = ' + str(Next))

    else:
        return newtonMethod(n + 1, Next)


newtonMethod(0, 2.0)  # 设置从开始计数，0x0 = 2.0
```

```
A = -0.48009375362057827,B = 1.4161468365471424,time = 0
Next x = 2.3390141059038383
A = 0.049067583543091375,B = 1.6948546583405764,time = 1
Next x = 2.3100631965726905
A = 0.0003041687020093331,B = 1.6737463368499903,time = 2
Next x = 2.3098814673014667
A = 1.2202990617993237e-08,B = 1.6736120345721872,time = 3
Next x = 2.3098814600100575
Meet f(x) = 0,x = 2.3098814600100575

Process finished with exit code 0
```

角度换算

$\frac{\theta}{2\pi} \times 360 = \frac{2.31}{2\pi} \times 360 = 132.35$度

5　成果展示

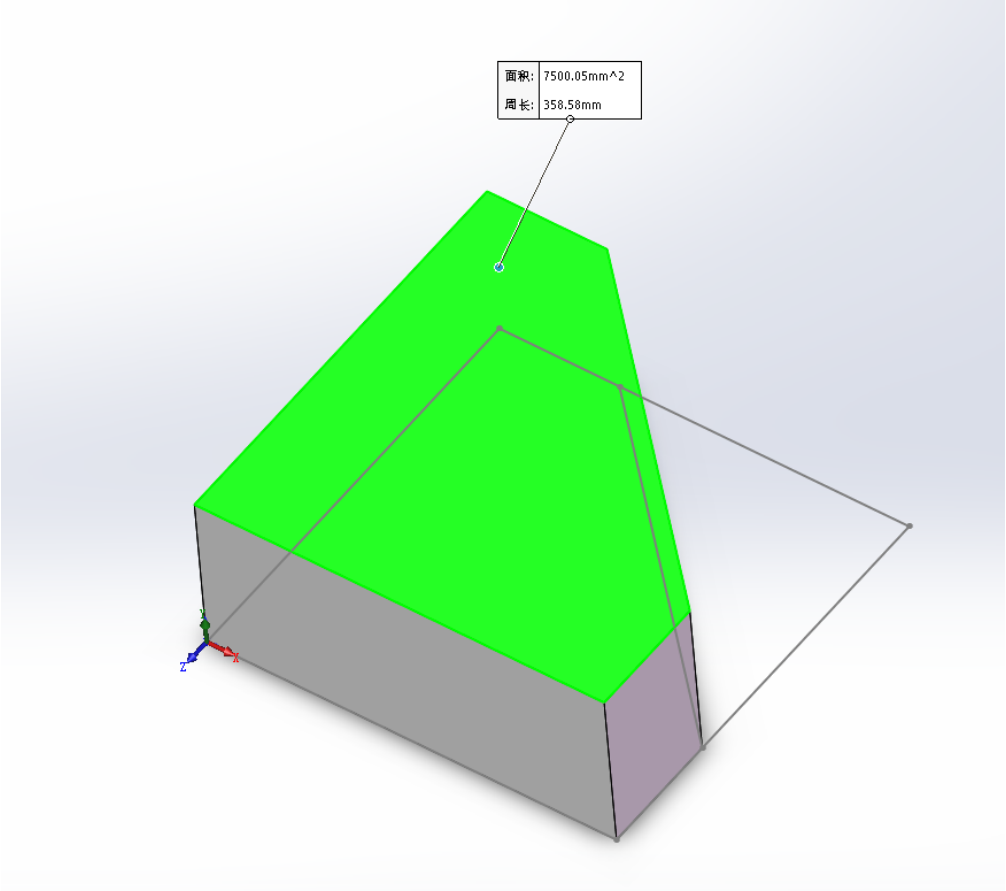

图 5: 正方形切蛋糕

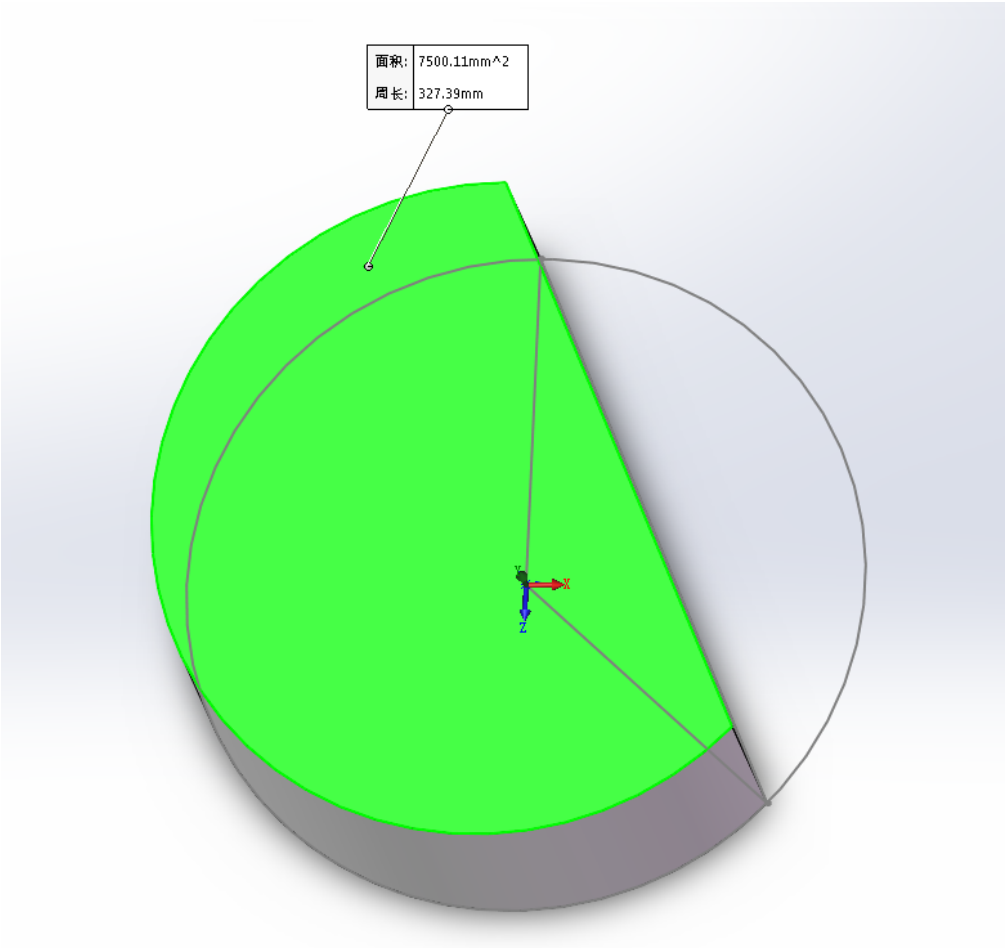

图 6: 圆形切蛋糕